

BENCHMARKS FOR CHOOSING A CURRICULUM ATTUNED TO THE STANDARDS FOR MATHEMATICAL PRACTICE

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EDC recently conducted a workshop for the state of Massachusetts entitled “Implementing the Standards for Mathematical Practice in Geometry, Algebra 2, and Precalculus.” Most of the week was taken up with finding ways to develop “everyday” high school mathematical activities (word problems, graphing, area formulas, factoring, and so on) in ways that help students experience the mathematical coherence and simplicity that comes from focusing on mathematical practice.

On the last day of the workshop, we spent some time looking at national reports, the upcoming MET revision from CBMS, and benchmarks for choosing a curriculum that claims to be aligned with the practice standards. The latter is quite relevant now; as many of the teachers reported, publishers are recycling previous editions with minor changes and “Common Core Compliant” stickers on the cover.

The following is adapted from a background piece I wrote as part of an advisory committee for the Massachusetts Commissioner of Education on exactly this topic of benchmarks for choosing a curriculum program. We discussed some of these recommendations at the workshop, and they resonated quite well with the participants.

Full disclosure: My group at EDC has published a 4-year high school curriculum, the *CME Project*, that is organized around mathematical habits of mind. It’s in use by about 30,000 students nationwide. We’re in the process of field testing a fifth course in the program: a high school course in linear algebra. We have an elementary program, *Think Math!*, based on the same principles. My colleagues and I have been working on the ideas that drive these programs for 4 decades. So, there’s no getting around it—everything that follows is intimately bound up with our work at EDC.

The first benchmark is the most important: **A curriculum should be explicit about its organizing principles.** This is far more important than how the program organizes and sequences topics, how it uses technology, how it treats contexts, or what pedagogy it employs. Evaluators of a program should not have to guess about a program’s intentions. These intentions should be stated up front and be clearly visible in every lesson and every problem set. In the coming months and years, publishers will be scurrying to “align” with CCSS and to get their products accepted by states and districts. There will be “cross-walks” (at a conference on Mathematics and the Common Core at UIC, Phil Daro called these “jaywalks”) that show where a program addresses individual content standards, and there will be all kinds of claims about the standards for mathematical practice. There are already vendors who will, for a price, evaluate programs for alignment with Common Core. My advice: *before looking at the details, ask the publisher or the authors to describe the the program’s underlying philosophy.* Especially important is the stance taken towards mathematical thinking and the practice standards.

Along these lines, it's essential that mathematicians are involved in a core way in any kind of evaluation process. Determining the extent to which a program exemplifies mathematical practices is best carried out by a practicing mathematician. Of *course* other parts of the mathematical community need to be involved in such deliberations—teachers are crucial to such discussions. But the practices capture the style of work used by mathematicians, and hence mathematicians need to look carefully at claims about how these are addressed.

There are some other features that, when present in a curriculum, help guarantee fidelity to mathematical practices. Here are some:

- **Experience before formality.** Worked-out examples and careful definitions are important, but students need to grapple with ideas and problems *before* these things are brought to closure. Definitions and theorems should be capstones, not foundations.
- **Textured emphasis.** A program with mathematical integrity makes a clear separation between convention and vocabulary on one hand and matters of mathematical substance on the other. Both kinds of things are important, but whether or not the positive y -axis is in the first quadrant is just not as important as the Pythagorean theorem. A careful look at the assessments provided with a curriculum tells a lot about whether or not the program has texture.
- **General purpose tools.** The methods and habits that students develop in high school should serve them well in their later work in mathematics and in their post-secondary endeavors. Beware of FOIL, the box method for factoring or setting up word problems, the $y = mx + b$ method for defining and graphing lines, and the dozens of other pieces of paraphernalia that have cluttered school mathematics for generations. None of this is in CCSS.
- **Technical fluency.** Expertise in numerical and algebraic calculation, proof, and graphing are essential in mathematics. A program that helps students develop habits of mind treats these essential skills in thoughtful ways. Mindless drills are not the same as carefully orchestrated etudes.
- **High expectations.** The vast majority of students have the capacity to think in ways that are characteristically mathematical. A curriculum that helps students develop mathematical practice features a small number of important themes, all developed with a low-threshold, high-ceiling design.
- **A mathematical community.** Writers, field testers, reviewers, and advisors should come from all parts of the mathematics community: teachers, mathematicians, education researchers, technology developers, and administrators. As I said above, it's especially important to get the involvement of practicing mathematicians.
- **Connect school mathematics to the discipline.** Every chapter, lesson, problem, and example should develop ideas that fit into the larger landscape of mathematics as a scientific discipline. Involving the entire mathematics community in the design of a program will help ensure that school mathematics does not remain a cultural enclave, cut off from mathematics as it is practiced after high school.